4.3 Logarithmic Functions

The inverse function of the exponential function with base b is called the ***logarithmic function with base b***.

Exponential Function: $f(x) = b^x$

Find the inverse:

1.) Replace $f(x)$ with y. $y = b^x$

2.) Interchange x and y. $x = b^y$

3.) Solve for y. $y = \log_b x$

Logarithmic Function

For $x > 0$ and $b > 0,\ b \neq 1$,

$y = \log_b x$ is equivalent to $b^y = x$.

***b*:** base ***x*:** argument

Ex. Write each equation in its equivalent exponential form or logarithmic form.

	Logarithmic Form	Exponential Form
a.)	$6 = \log_2 64$	
b.)	$2 = \log_9 x$	
c.)	$\log_5 125 = y$	
d.)		$5^{-3} = \frac{1}{125}$
e.)		$\sqrt[3]{64} = x$
f.)		$b^3 = 343$
g.)		$8^y = 300$

Ex. Evaluate each expression without using a calculator.

a.) $\log_7 49 =$

b.) $\log_3 \frac{1}{27} =$

c.) $\log_6 \sqrt{6} =$

Special Logarithms

Common Logarithm (base 10) $\log_{10} x = \log x$

Natural Logarithm (base e) $\log_e x = \ln x$

Basic Properties of Logarithms

1.) $\log_b 1 = 0 \iff \log 1 = 0 \iff \ln 1 = 0$

2.) $\log_b b = 1 \iff \log 10 = 1 \iff \ln e = 1$

3.) $\log_b b^x = x \iff \log 10^x = x \iff \ln e^x = x$

4.) $b^{\log_b x} = x \iff 10^{\log x} = x \iff e^{\ln x} = x$

Ex. Evaluate each expression without using a calculator.

a.) $\log_{11} 11 =$

b.) $\log_\pi 1 =$

c.) $\ln e =$

d.) $\log 10^8 =$

e.) $10^{\log\left(a^2+3\right)} =$

f.) $7^{\log_7 22} =$

g.) $\ln \dfrac{1}{e^7} =$

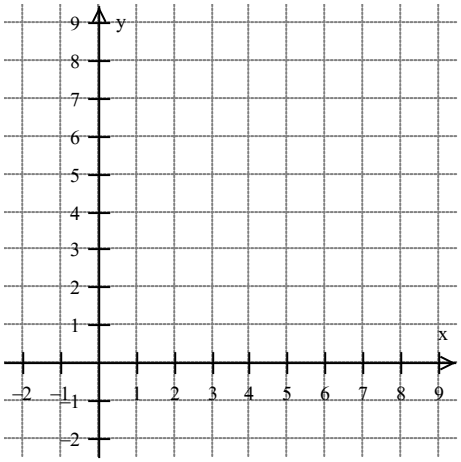
h.) $e^{\ln 7x^2} =$

❖ **Graphs of Logarithmic Functions**

Ex. Graph $f(x) = 3^x$ and $g(x) = \log_3 x$ in the same rectangular coordinate system.

x	$y = 3^x$
-2	
-1	
0	
1	
2	

x	$y = \log_3 x$



Domain of f : _____

Range of f : _____

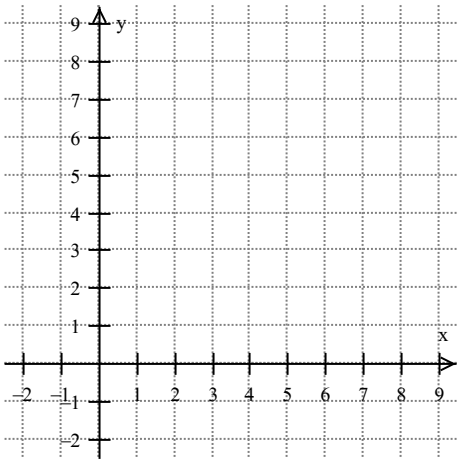
Domain of g : _____

Range of g : _____

Ex. Graph $f(x)=\left(\frac{1}{2}\right)^x$ and $g(x)=\log_{\frac{1}{2}} x$ in the same rectangular coordinate system.

x	$y = (1/2)^x$
-2	
-1	
0	
1	
2	

x	$y = \log_{(1/2)} x$



Domain of f: _____ Domain of g: _____

Range of f: _____ Range of g: _____

Characteristics of Logarithmic Graphs of the Form $f(x)=\log_b x$: (p.434)

1) Domain: _____

   Range: _____

2) The point that all graphs pass through: _____

   x-intercept: _____

   y-intercept: _____

3) $b>1$: $f(x)=\log_b x$ is an _____ function

4) $0<b<1$: $f(x)=\log_b x$ is an _____ function

5) Vertical Asymptote: _____

Note:
Exponential function always has a horizontal asymptote.
Logarithmic function always has a vertical asymptote.

❖ The Domain of a Logarithmic Function

The domain of a logarithmic function, $f(x)=\log_b x$, is the set of all _____ real numbers.

In general, the domain of $f(x)=\log_b g(x)$ consists of all x for which $g(x)>0$.

Ex. Find the domain of each logarithmic function in interval notation.

a) $f(x)=\log_5(x+6)$

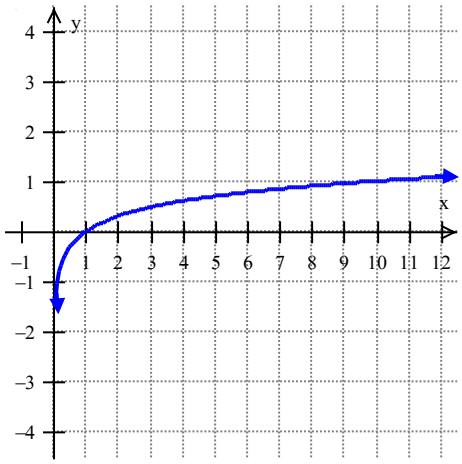
b) $f(x)=\ln(7-3x)$

❖ Transformations of Logarithmic Functions

Ex. Given the graph of $f(x)=\log x$.

i) Use the transformations of this graph to graph the given function.
ii) Give equations of the asymptotes.
iii) Use the graphs to determine each function's domain and range.

(a) $g(x)=\log(x-2)$

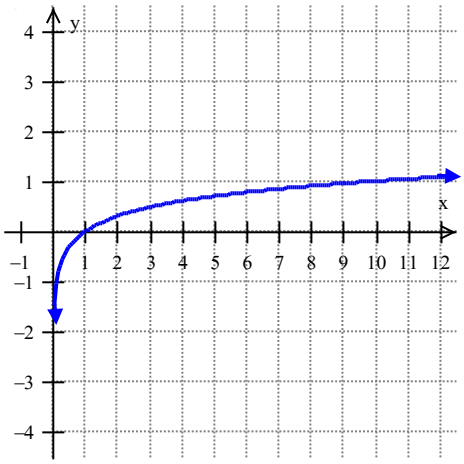


V.A.: _____

Domain: _____

Range: _____

(b) $h(x)=2-\log x$



V.A.: _____

Domain: _____

Range: _____